Studies on Structural Monitoring and Identification

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Contents

- Background: Condition assessment, Ultimate state and Performance maintenance of ancient timber structure
- Research: Studies on Structural System Identification and Optimal Sensor Placement Methods in Time Domain

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Ancient timber building is of great value

Ancient Timber building is an important part of Chinese architecture which has the largest proportion in ancient buildings. Many ancient Timber buildings are world-renowned national treasures which are of historical, religious and artistic values.

Structural safety problem of ancient timber building

Most of the ancient buildings are suffering from different degrees of structural damages because of environmental effects, earthquake, material degradation, etc.

Multiple loads acting on the structure

Wind, snow, gravity and other natural loads

Crowd loads

Earthquakes and other incidental loads

Traffic loads

Basic principles of structural maintenance

- Protection first
- Service life extension

- Whether the structure needs maintenance or not
- When will the maintenance be carried out

Condition assessment

- What are the ultimate states of the structure before and after maintenance is taken place

Ultimate states

- How to maintain the structure
- How to improve existing maintenance techniques

Performance maintenance

Research Contents

1. Mechanical properties of ancient timber structure

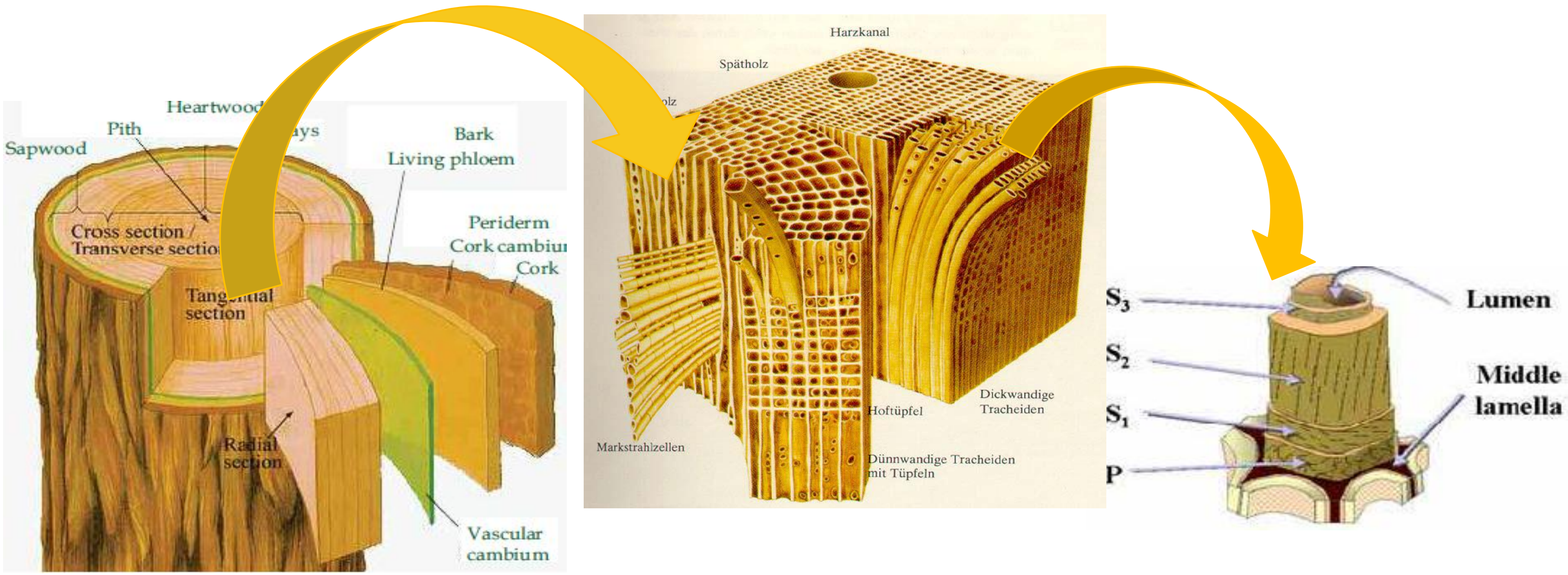
2. Condition assessment of ancient timber structure

3. Ultimate states of ancient timber structure

4. Performance maintenance of ancient timber structure

Mechanical properties

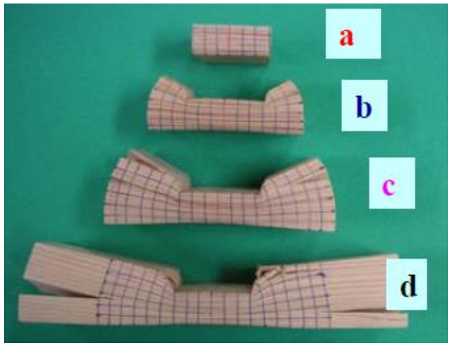
- Timber is a kind of porous biological material.

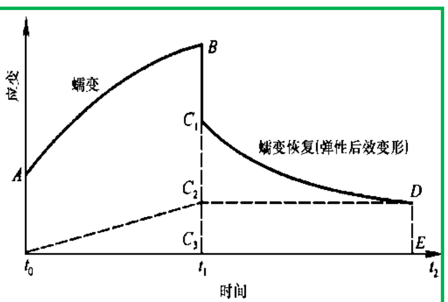


Viscoelastic Hygroscopic Orthotropic …

Mechanical properties

- Characteristics of timber material





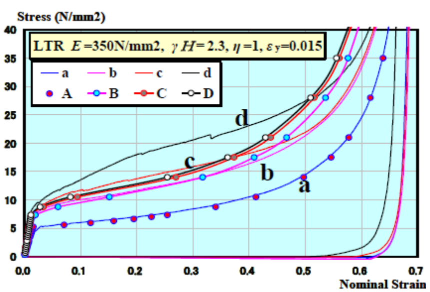


Fig.6 Stress-strain diagram of large strain embedment tests

Varying with time

Volume variation

- Characteristics of joints

Scaled-model

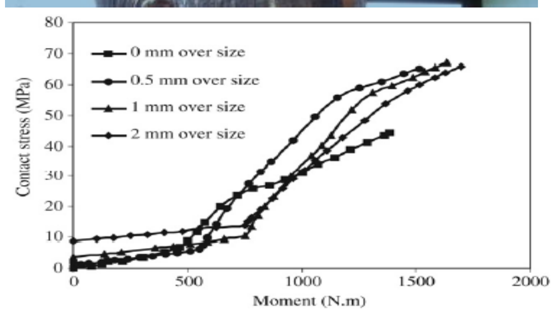


Fig. 12. Contact stress in the beam versus moment.

Error amplification

Non-linear contact between components

Semi-rigid joint

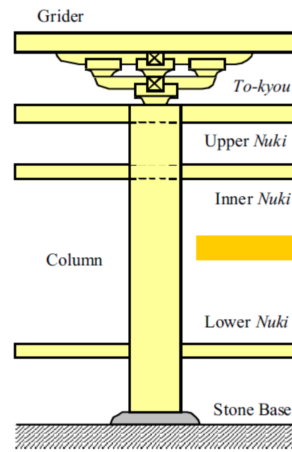
Mechanical properties

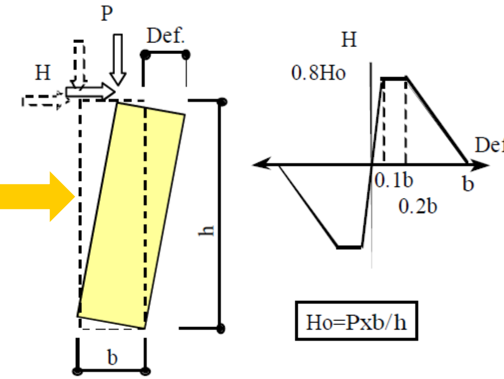
- The material model of the old timber based on test results.
- The joint model based on full-scale model tests.
- The finite element model of the whole structure based on the previously established material and joint models.

（a）柱头铺作模型

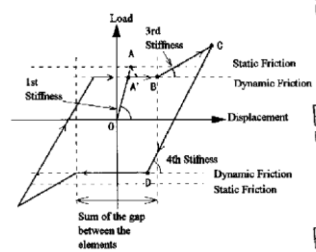
（b）补间铺作模型

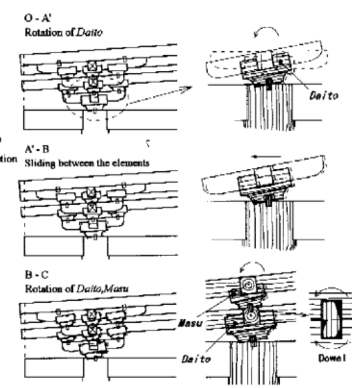
Dou Gong

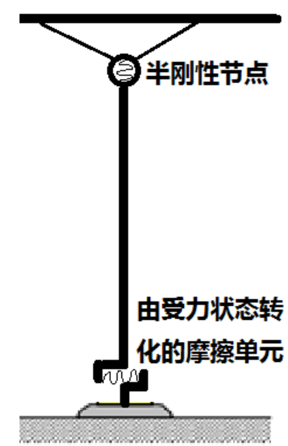




Rotation of column







Analyze the connection between Dougong and column

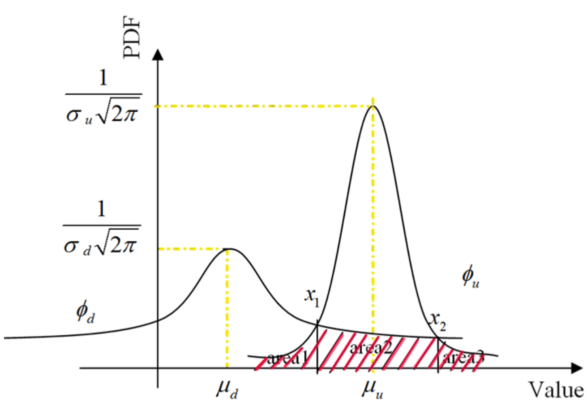
Condition Assessment

- Multiple uncertainties

Physical parameter | Boundary condition | Joint parameter

Low identification accuracy

Probabilistic damage identification method

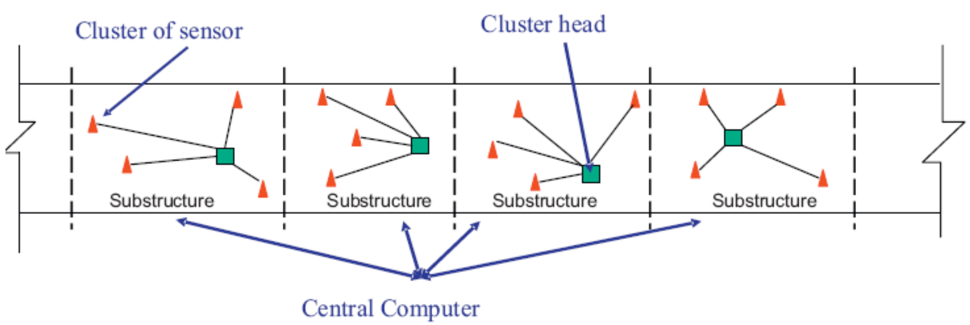


- System with weak connections

Big contact damping | Small distance of wave transmission

Dynamic loads have big effects on the loading area and have quite small effects on the un-loading area

Substructure damage identification method



Condition Assessment

- Principles for sensor placement

Less perturbation — Protection first

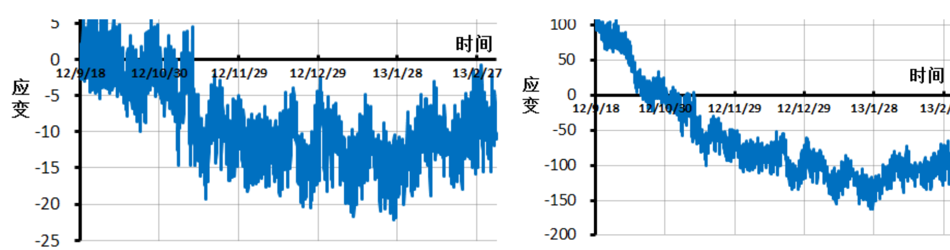
More constraints for sensor placement

- Influence of environment effects

Timber is sensitive to temperature and humidity

Variations of the structure parameter caused by environmental effects are sometime bigger than those caused by damages

De-coupling of the environmental factors

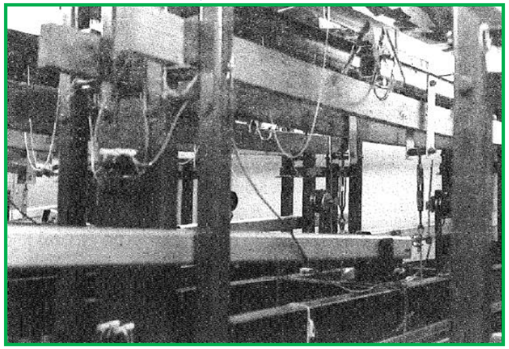


Ultimate State

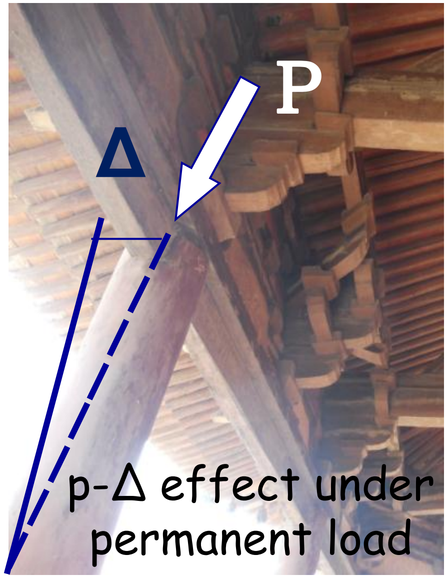
- **Ageing model of the ancient timber structure**

Most ageing models are for timber materials, and there are few researches for the ageing model of the whole timber structure. It is important to establish the ageing model of the timber frame based on the material model previously established and site survey results .

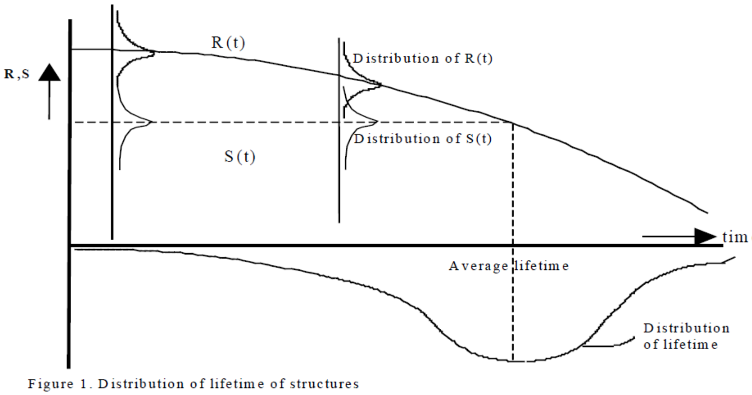
Ageing mode of components



Test with sustained loads



Ageing model of the whole structure



Full life model of the timber frame

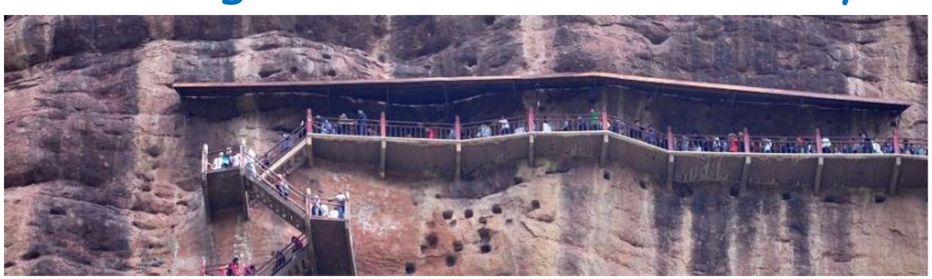
Ultimate State

- The ultimate states under permanent load, earthquake, and wind load.

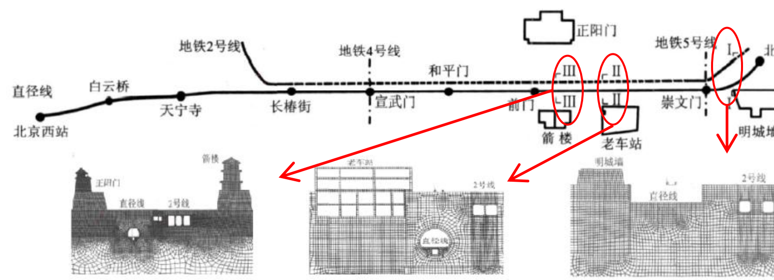
Determine the ultimate bearing capacity of the ancient timber structure under permanent load, earthquake and wind load based on the ageing model; analyze the energy potential and energy distribution mechanism of the ancient timber structure.

- The ultimate states under traffic, crowd and any other controllable loads

Determine the limit states under traffic and crowd loads, and reduce the vibration of the structure by limiting the number of visitors, controlling the value of loads, or by other technical methods.



The alarm value of the crowd load of the wooden bridge



The influence of the subway train to the ancient building

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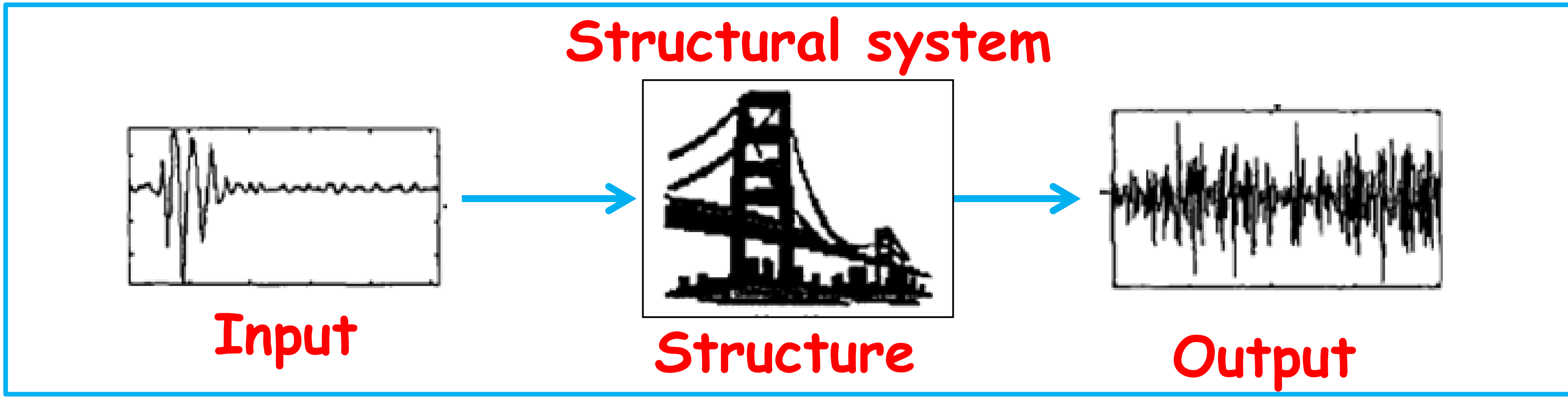
Structural System Identification



Structural system identification

- Force identification
  - Structure + Output ⟶ Input
- Structural parameter identification
  - Output + Input ⟶ Structure
  - Comparison between two structural states ⟶ Damage identification

Problems and difficulties in:

- **Sensor placement method**

To obtain the output, site tests should be carried out first. Non-proper sensor placement will lead to bad identification results, or even , the structural properties can not be identified at all.

- Structural identification is a kind of ill-conditioned inverse problem. The conditioning of the identification equation, relating to sensor placement, has great influences on the identification accuracy. Existing sensor placement methods seldom consider about this fact.

↓

Sensor placement method based on the conditioning analysis of the identification equation.

Sensor placement method based on the conditioning analysis of the identification equation

Sensor placement method based on the conditioning analysis of the identification equation

Take force identification in state space as the research background

(Kammer, 1992; Mao, 2010; Law, 2011)

The equation of motion of the structural system can be expressed in the state space as following

Unknown

$$\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z} + \mathbf{B}\mathbf{L}\mathbf{f}(t)$$ ----State equation

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix} \qquad \mathbf{z}(t) = \begin{Bmatrix} \mathbf{x}(t) \\ \dot{\mathbf{x}}(t) \end{Bmatrix} \qquad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1} \end{bmatrix}$$

State variable

If responses at all DOFs are known, the unknown forces can be calculated. However, in practice, only responses at some measured DOFs are known.

The above equation can be converted into the following discrete equation as

$$\mathbf{z}(t_{j+1}) = \mathbf{A}^D \mathbf{z}(t_j) + \mathbf{B}^D \mathbf{f}(t_j)$$

$$\mathbf{A}^D = \exp(\mathbf{A}\Delta t) \qquad \mathbf{B}^D = \mathbf{A}^{-1}(\exp(\mathbf{A}dt) - \mathbf{I})\mathbf{B}$$

$$\mathbf{z}(t_{j+1}) = \mathbf{A}^D \mathbf{z}(t_j) + \mathbf{B}^D \mathbf{f}(t_j)$$

Denote vector **y** to represent the output(measured responses) of the structural system and it is assembled from the measurements with

Relating to sensor placement

$$\mathbf{y} = \mathbf{R_a}\ddot{\mathbf{x}} + \mathbf{R_v}\dot{\mathbf{x}} + \mathbf{R_d}\mathbf{x}$$ ----Output

R_a, R_v and R_d are the output influence matrices for the measured acceleration, velocity and displacement respectively.

y can be represented by the state variable

$$\mathbf{y} = \mathbf{Rz} + \mathbf{DLf}$$ ----Observation equation

$$\mathbf{R} = \left[\mathbf{R_d} - \mathbf{R_a}\mathbf{M}^{-1}\mathbf{K} \quad \mathbf{R_v} - \mathbf{R_a}\mathbf{M}^{-1}\mathbf{K} \right] \qquad \mathbf{D} = \mathbf{R_a}\mathbf{M}^{-1}$$

and can be converted into the following discrete equation as

$$\mathbf{y}(j) = \mathbf{Rz}(j) + \mathbf{DLf}(j)$$

Assuming **zero initial response of the structure**, the output of the system $\mathbf{y}(j)$ can be obtained from the discrete state equation and observation equation in terms of the previous input $\mathbf{f}(k)$ $(k=0,1,\ldots,j)$

$$\mathbf{y}(j)=\mathbf{DLF}(j)+\sum_{k=1}^{j}\mathbf{R}\left(\mathbf{A_d}\right)^{k\text{-}1}\mathbf{B_d}\mathbf{L}\mathbf{f}\left(j-k\right)$$

Let $\mathbf{H}_0=\mathbf{DL}$ and $\mathbf{H}_k=\mathbf{R}\left(\mathbf{A}_d\right)^{k-1}\mathbf{B}_d\mathbf{L}$

$$\mathbf{y}(j)=\sum_{k=0}^{j}\mathbf{H}_k\mathbf{f}(j-k)$$

Ill-conditioning inverse equation

$$\mathbf{y}=\mathbf{Hf}$$

Force identification equation

the Markov parameter matrix

$$\mathbf{y}_{(N_m\times n)\times 1}=\begin{bmatrix}\mathbf{y}(0)\\ \mathbf{y}(1)\\ \vdots\\ \mathbf{y}(n-1)\end{bmatrix},\quad \mathbf{H}_{(N_m\times n)\times(N_f\times n)}=\begin{bmatrix}\mathbf{H}_0 & 0 & \cdots & 0\\ \mathbf{H}_1 & \mathbf{H}_0 & \cdots & 0\\ \vdots & \vdots & \ddots & 0\\ \mathbf{H}_{n-1} & \mathbf{H}_{n-2} & \cdots & \mathbf{H}_0\end{bmatrix},\quad \mathbf{f}_{(N_f\times n)\times 1}=\begin{bmatrix}\mathbf{f}(0)\\ \mathbf{f}(1)\\ \vdots\\ \mathbf{f}(n-1)\end{bmatrix}$$

Solution with regularization method

Method usually used in solving the inverse problems

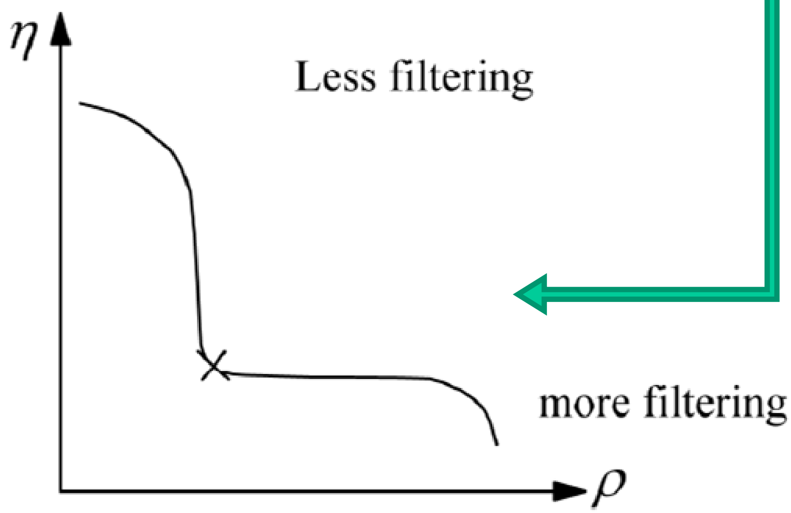
- **Tikhonov Regularization method**

Cost function: $\min\left\{\left\|\mathbf{H}\mathbf{f}_{\text{reg}}-\mathbf{y}\right\|_2^2+\lambda^2\left\|\mathbf{f}_{\text{reg}}\right\|_2^2\right\}$

Least-squares solution ρ — Side constraint η

Regularized solution: $\mathbf{f}_{\text{reg}}=[\mathbf{H}^{\text{T}}\mathbf{H}+\lambda^2\mathbf{I}]^{-1}\mathbf{H}^{\text{T}}\mathbf{y}$

Regularization parameter



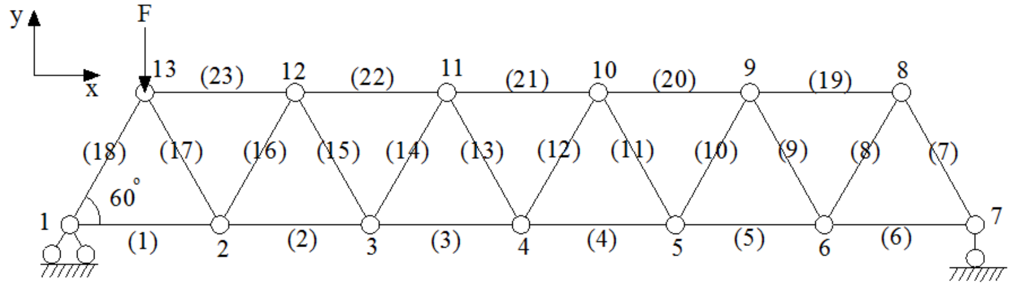
- The L-curve method (Hansen 1992) is used for determining the regularization parameter.
- L-curve is a plot on all valid regularization parameters with the norm of the regularized solution versus the corresponding residual.
- The point with maximum curvature in the L-curve gives a compromise of ρ and η, and the corresponding λ is selected to be the regularization parameter.

Problems in application

5% model errors and 5% measurement noise

Simulation with a truss structure

$$\mathbf{F}(t) = 10\sin(30\pi t + 1.4\pi) + 10\sin(60\pi t) + 8\sin(90\pi t + 0.9\pi) \qquad N$$



Number of sensor	Sensor location	Cond(H)	Error(%)		
			Mean	Standard deviation	Peak
1	2(y)	194.00	11.08	3.31	20.67
2	2(y),12(x)	193.93	8.77	2.41	15.97
3	2(y),4(y),12(x)	233.45	8.92	2.44	18.26
4	2(y),4(y),12(x),12(y)	208.44	8.88	8.92	17.46
1	11(y)	5.15×10^{16}	67.57	44.65	198.84
2	3(y),11(y)	616.17	27.31	6.27	45.95
3	3(y),6(x),11(y)	635.73	27.53	7.30	52.38
4	3(y),6(x),11(x),11(y)	636.26	28.01	7.80	51.92

- If there is no model error or measurement noise, the identification error are quite small (close to zero)for all cases.
- Different sensor locations will lead to different identification errors with consideration of model errors and measurement noise;
- Some sensor placement will lead to quite low identification accuracy even using the regularization method.

Sensor placement method for force identification

Perturbation analysis

Perturbation in the model (model error)

$$\mathbf{y} = (\mathbf{H} + \Delta\mathbf{H})(\mathbf{f} + \Delta\mathbf{f})$$

$$-\Delta\mathbf{f} = \mathbf{H}^{-1}\Delta\mathbf{H}\mathbf{f} + \mathbf{H}^{-1}\Delta\mathbf{H}\Delta\mathbf{f}$$

$$\frac{\|\Delta\mathbf{f}\|}{\|\mathbf{f}\|} \le \frac{\|\mathbf{H}\|\|\mathbf{H}^{-1}\|}{1 - \|\Delta\mathbf{H}\|\|\mathbf{H}^{-1}\|} \frac{\|\Delta\mathbf{H}\|}{\|\mathbf{H}\|}$$

Perturbation in the response (measurement noise)

$$\mathbf{y} + \Delta\mathbf{y} = \mathbf{H}(\mathbf{f} + \Delta\mathbf{f})$$

$$\Delta\mathbf{f} = \mathbf{H}^{-1}\Delta\mathbf{y}$$

$$\frac{\|\Delta\mathbf{f}\|}{\|\mathbf{f}\|} \le \left(\|\mathbf{H}\|\|\mathbf{H}^{-1}\|\right) \frac{\|\Delta\mathbf{y}\|}{\|\mathbf{y}\|}$$

$$cond(\mathbf{H}) = \|\mathbf{H}\|\|\mathbf{H}^{-1}\|$$

Condition number

Condition number is a definition in matrix computation theory. The larger the condition number is ,the more ill-conditioning the equation is.

$$\mathbf{H}\mathbf{f} = \mathbf{y}$$

The condition number of the Markov parameter matrix can be a measure reflecting the conditioning of the identification equation.

Sensor placement method

Method I : CN method

Based on the condition number of Markov parameter matrix

$$\mathbf{Hf} = \mathbf{y}$$

Smallest ill-conditioning

→

$$cond(\mathbf{H}) = \|\mathbf{H}\| \|\mathbf{H}^{-1}\|$$

Smallest condition number

→

Optimal sensor placement

Detailed procedure:

i. Determine the number of sensor and the number of candidate sensor location.

ii. Compute all possible combinations of responses.

iii. Calculate the system Markov parameter matrix for all the above combinations.

iv. The combination giving the minimum condition number is taken as the optimal combination of sensor locations.

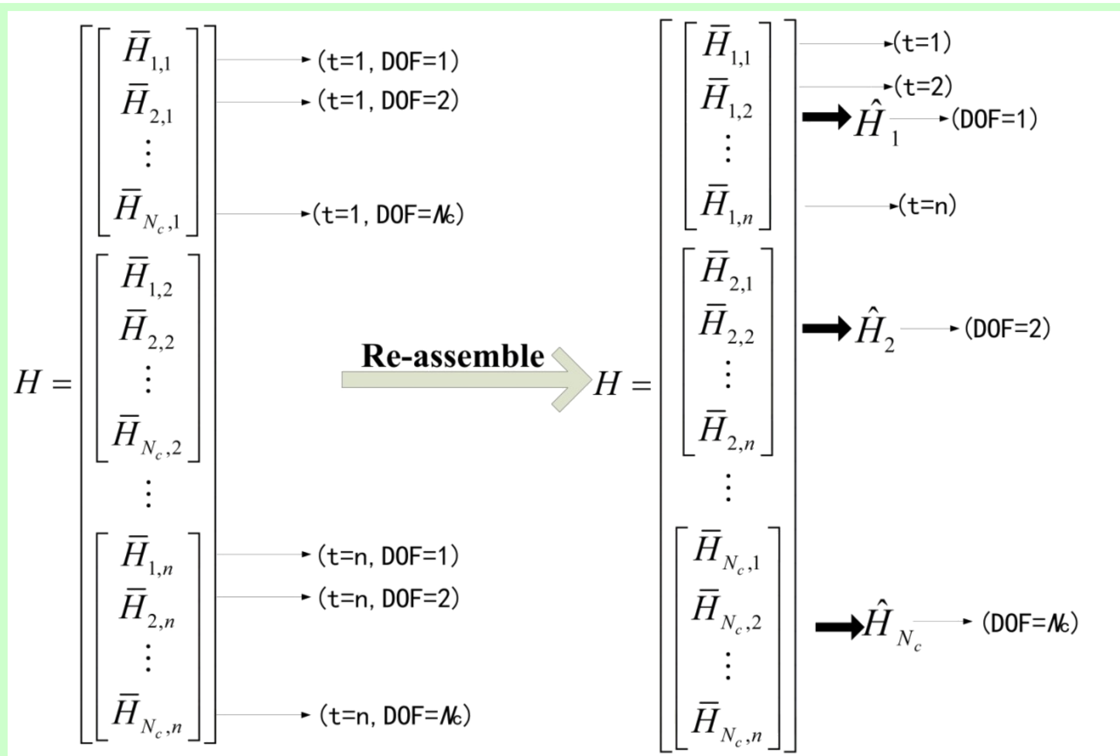
cumbrous

However, the process would involve the estimation of condition number of all possible sensor combinations. This task becomes impractical with the increasing of the number of candidate combinations of sensor locations.

Method II : CA method----can be an alternative approach to Method I

Based on the correlation analysis of Markov parameter matrix

Structural responses can be decomposed as the sum of a series of independent components. If the measured data of each sensor may represent an independent component, the accuracy will be the best, i.e., the correlation of different row vectors of H should be low.



Factors which influence H

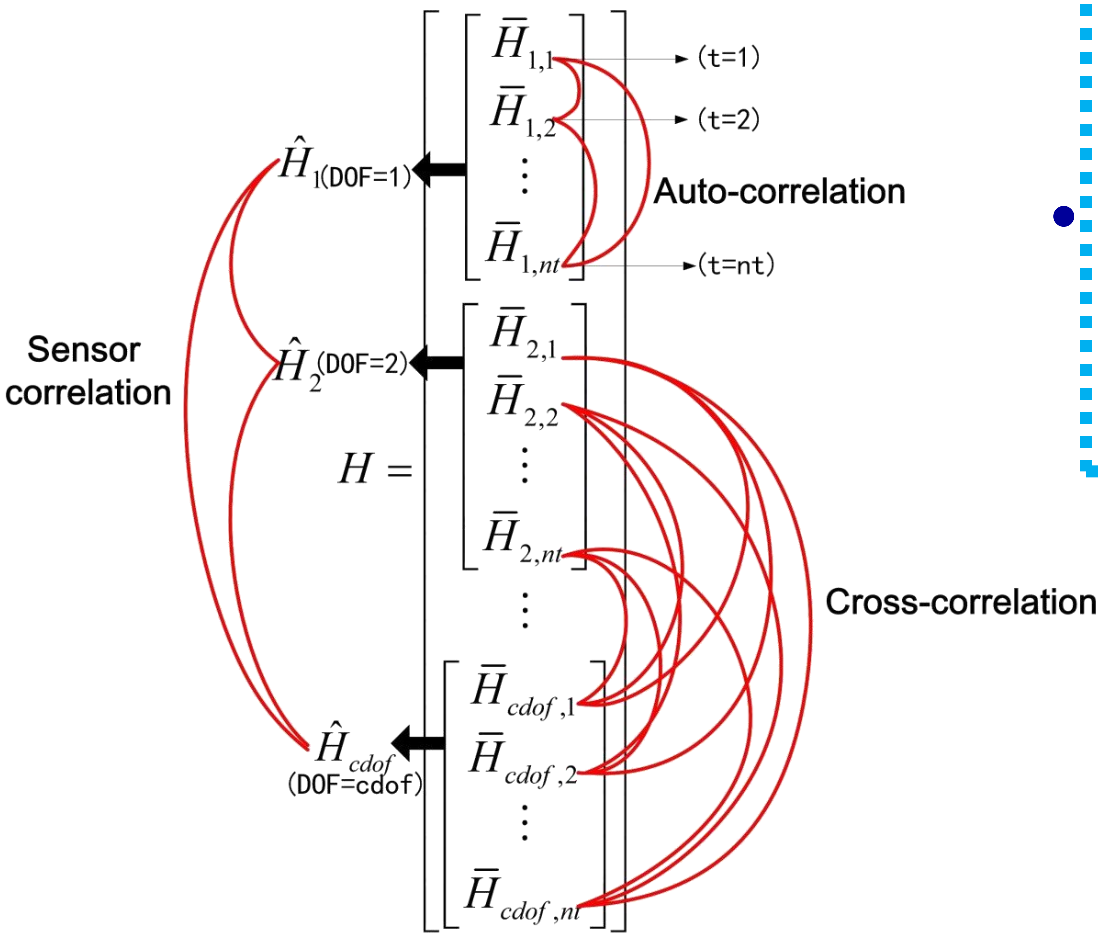
- Structure properties
- Sensor location
- Sampling frequency and time
- Location of external forces

Require not much computation effort

independence of sensor responses → correlation of row vectors of matrix H → Conditioning of matrix H

Definition of correlations:

Sensor combination with Smallest sensor correlation ⇒ Optimal sensor placement



- **Sensor correlation:** correlation between different sensor locations.
- **Auto correlation:** correlation between different sampling points for one certain sensor location.
- **Cross correlation:** correlation between different sampling points for different sensor locations.

⇕

Determined by the correlation of row vectors of matrix H

Sensor correlation matrix:

- Nc- number of candidate sensor location
- Nm- number of sensor

Sensor correlation matrix for all of the candidate sensor locations

$$\mathbf{R}=\begin{bmatrix} r_{1,1} & r_{1,2} & \cdots & r_{1,p} & \cdots & r_{1,N_c} \\ r_{2,1} & r_{2,2} & \cdots & r_{2,p} & \cdots & r_{2,N_c} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ r_{p,1} & r_{p,2} & \cdots & r_{p,p} & \cdots & r_{p,N_c} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{N_c,1} & r_{N_c,2} & \cdots & r_{N_c,p} & \cdots & r_{N_c,N_c} \end{bmatrix}_{(N_c \times N_c)}$$

Determined by cross correlation matrix

Determined by Auto correlation matrix

Full matrix

extract

Sub matrix

Sensor correlation matrix for a certain sensor combination

$$\mathbf{R}=\begin{bmatrix} r_{1,1} & r_{1,2} & \cdots & r_{1,N_m} \\ r_{2,1} & r_{2,2} & \cdots & r_{1,N_m} \\ \vdots & \vdots & \ddots & \vdots \\ r_{N_m,1} & r_{N_m,2} & \cdots & r_{N_m,N_m} \end{bmatrix}_{(N_m \times N_m)}$$

Cross-correlation matrix

$$\mathbf{E}(p,q)=\begin{bmatrix} e_{1,1} & e_{1,2} & \cdots & e_{1,n} \\ e_{1,2} & e_{2,2} & \cdots & e_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ e_{1,n} & e_{2,n} & \cdots & e_{n,n} \end{bmatrix}_{(n\times n)}$$

$$r_{p,q}=\max\left\{e_{l_1,l_2}(p,q)\right\}$$

$$e_{l_1,l_2}(p,q)=\frac{\left|\bar{\mathbf{H}}_{p,l_1}\bar{\mathbf{H}}^{\mathrm{T}}_{q,l_2}\right|}{\sqrt{\left|\bar{\mathbf{H}}_{p,l_1}\bar{\mathbf{H}}^{\mathrm{T}}_{p,l_1}\right|}\sqrt{\left|\bar{\mathbf{H}}_{q,l_2}\bar{\mathbf{H}}^{\mathrm{T}}_{q,l_2}\right|}}$$

$$\mathbf{R}=\begin{bmatrix} r_{1,1} & r_{1,2} & \cdots & r_{1,p} & \cdots & r_{1,N_c} \\ r_{2,1} & r_{2,2} & \cdots & r_{2,p} & \cdots & r_{2,N_c} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ r_{p,1} & r_{p,2} & \cdots & r_{p,p} & \cdots & r_{p,N_c} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{N_c,1} & r_{N_c,2} & \cdots & r_{N_c,p} & \cdots & r_{N_c,N_c} \end{bmatrix}_{(N_c\times N_c)}$$

Non-diagonal element in the sensor correlation matrix

Auto-correlation matrix

$$\mathbf{S}(p)=\begin{bmatrix} 0 & s_{1,2} & \cdots & s_{1,n} \\ s_{1,2} & 0 & \cdots & s_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ s_{1,n} & s_{2,n} & \cdots & 0 \end{bmatrix}_{(n\times n)}$$

$$r_{p,p}=\max\left\{s_{l_1,l_2}(p)\right\}$$

$$s_{l_1,l_2}(p)=\frac{\left|\bar{\mathbf{H}}_{p,l_1}\bar{\mathbf{H}}^{\mathrm{T}}_{p,l_2}\right|}{\sqrt{\left|\bar{\mathbf{H}}_{p,l_1}\bar{\mathbf{H}}^{\mathrm{T}}_{p,l_1}\right|}\sqrt{\left|\bar{\mathbf{H}}_{p,l_2}\bar{\mathbf{H}}^{\mathrm{T}}_{p,l_2}\right|}}$$

$$\mathbf{R}=\begin{bmatrix} r_{1,1} & r_{1,2} & \cdots & r_{1,p} & \cdots & r_{1,N_c} \\ r_{2,1} & r_{2,2} & \cdots & r_{2,p} & \cdots & r_{2,N_c} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ r_{p,1} & r_{p,2} & \cdots & r_{p,p} & \cdots & r_{p,N_c} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{N_c,1} & r_{N_c,2} & \cdots & r_{N_c,p} & \cdots & r_{N_c,N_c} \end{bmatrix}_{(N_c\times N_c)}$$

Diagonal element in the sensor correlation matrix

Sensor correlation criterion

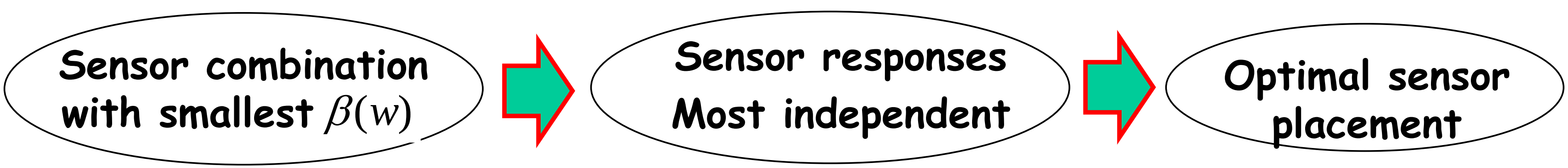
$$\beta(w) = \left\| R_{(N_m \times N_m)} \right\|_F = \left(\sum_{k_1=1}^{N_m} \sum_{k_2=1}^{N_m} r_{k_1 k_2}^2 \right)^{1/2}$$

$$\left(w = 1, 2, \cdots C_{N_c}^{N_m} \right)$$

Total number of sensor combination

Element in sensor correlation matrix for a certain sensor combination

$$\mathbf{R} = \begin{bmatrix} r_{1,1} & r_{1,2} & \cdots & r_{1,N_m} \\ r_{2,1} & r_{2,2} & \cdots & r_{1,N_m} \\ \vdots & \vdots & \ddots & \vdots \\ r_{N_m,1} & r_{N_m,2} & \cdots & r_{N_m,N_m} \end{bmatrix}_{(N_m \times N_m)}$$



Numerical simulation

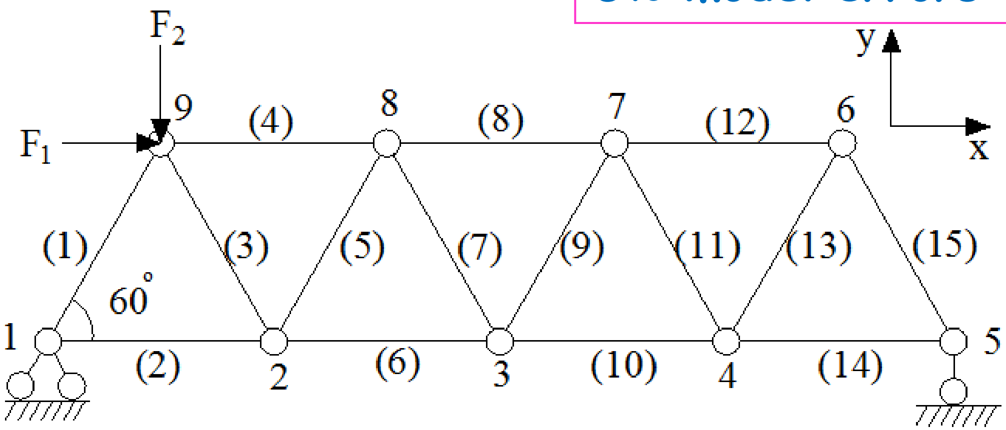
- Compare the two proposed sensor placement methods;
- Demonstrate the effectiveness of the alternative CA method.

Truss structure

$$\mathbf{F}_1(t) = 80\sin(50\pi t + 0.5\pi) + 50\sin(80\pi t) + 10\sin(120\pi t + 0.6\pi) \quad N$$

$$\mathbf{F}_2(t) = 60\sin(55\pi t) + 40\sin(70\pi t + 1.4\pi) + 15\sin(100\pi t) \quad N$$

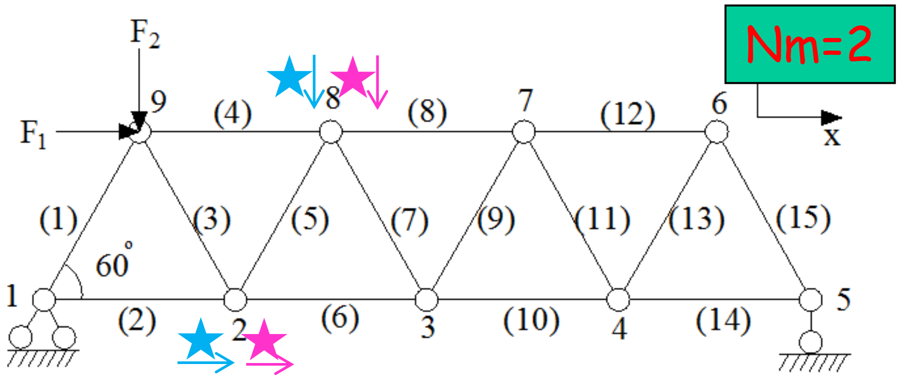
5% model errors and 5% measurement noise

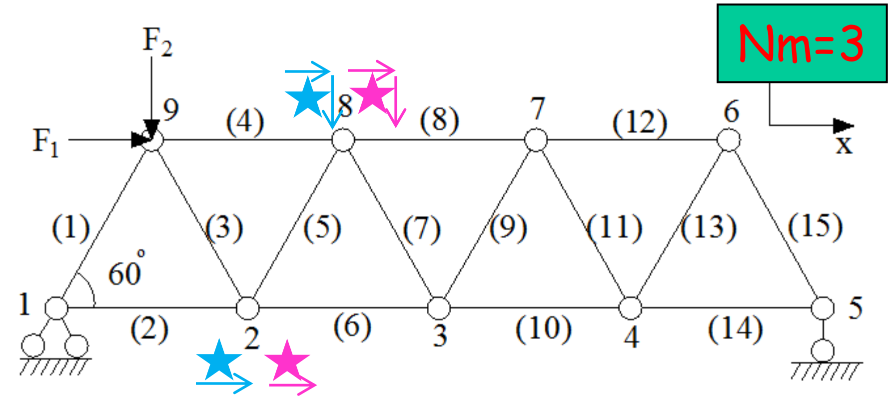


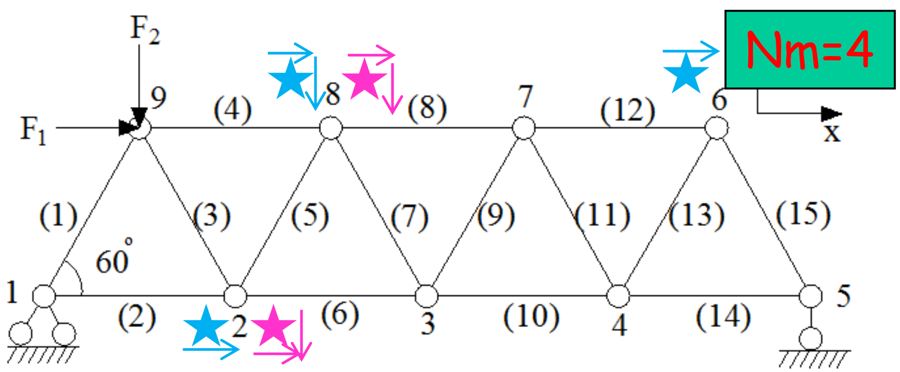
CA method has a great advantage with the computation efficiency.

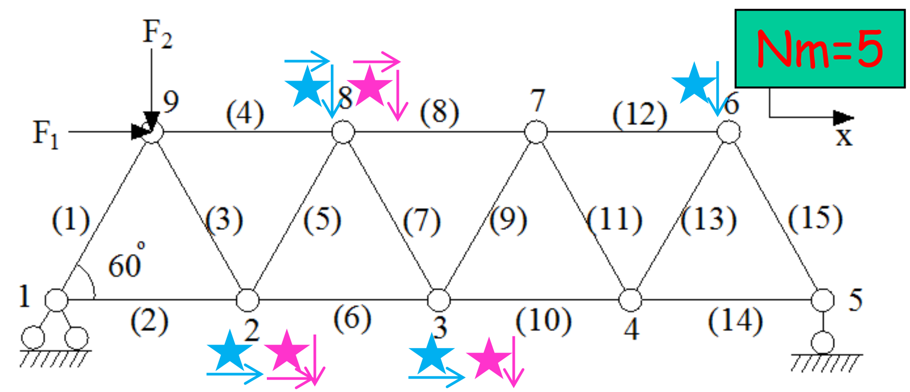
Number of sensors	Number of candidate combination of sensor locations	Computing time (s)	
		Condition number (CN) method	Correlation analysis (CA) method
2	$C_{12}^2 = 66$	65.14	124.09
3	$C_{12}^3 = 220$	268.40	126.26
4	$C_{12}^4 = 495$	693.17	124.66
5	$C_{12}^5 = 792$	1330.65	126.96
6	$C_{12}^6 = 924$	1653.96	126.74

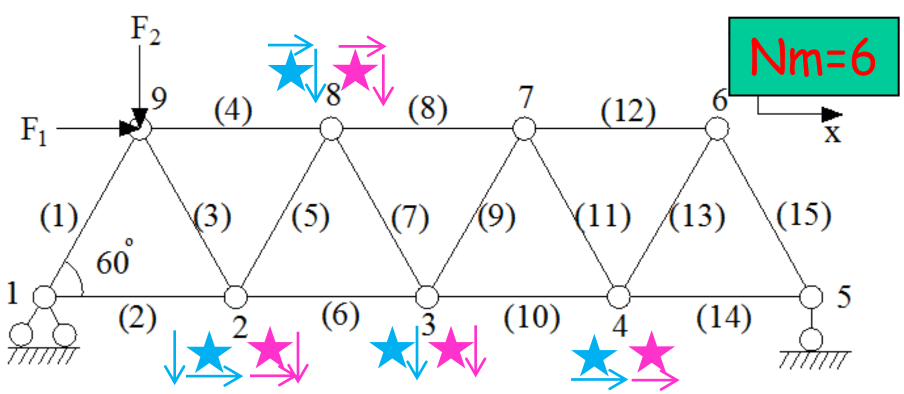
Comparison of optimal sensor placement with different number of sensor











★ CN ★ CA

Location in the optimal sensor combination, x direction

Location in the optimal sensor combination, y direction

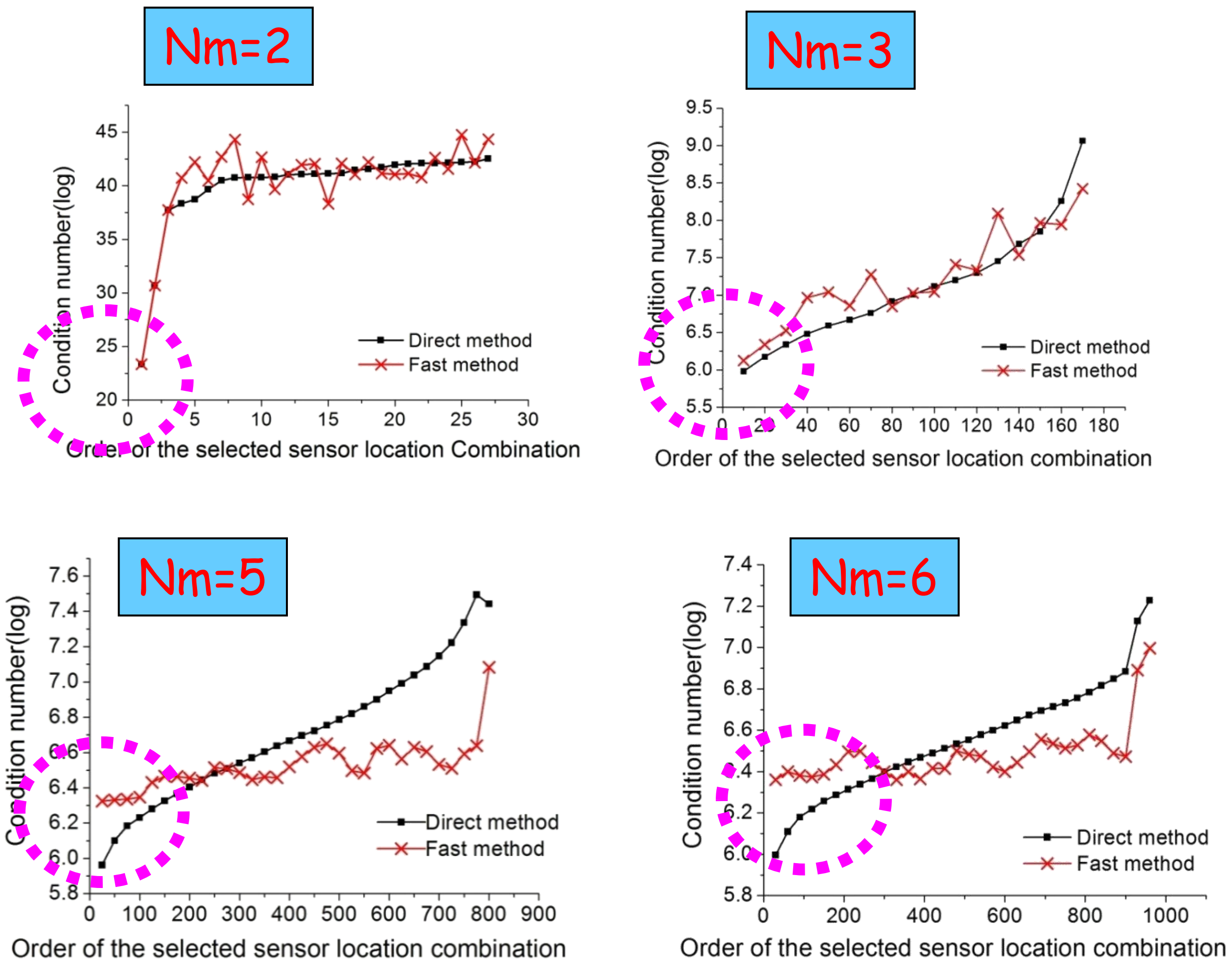
The locations in the optimal sensor combination are close to the location of external forces.

Type	Method	Number of sensor	Location	Condition number
Optimal Combination	CN	2	N2(x), N8(y)	1.35×10^{10}
		3	N2(x), N8(x), N8(y)	326.10
		4	N2(x), N6(x), N8(x), N8(y)	329.74
		5	N2(x), N3(x), N6(y), N8(x), N8(y)	337.48
		6	N2(x), N3(x), N4(y), N6(y), N8(x), N8(y)	345.91
	CA	2	N2(x), N8(y)	1.35×10^{10}
		3	N2(x), N8(x), N8(y)	326.10
		4	N2(x), N2(y), N8(x), N8(y)	389.76
		5	N2(x), N2(y), N3(y), N8(x), N8(y)	536.90
		6	N2(x), N2(y), N3(y),N4(x),N8(x), N8(y)	563.18

- When the number of sensor is less than or equal to 3, the optimal sensor location combinations selected by the two methods are the same. The condition number increases slightly with an increase of number of sensors for both methods.
- When the number of sensor is 4 or larger, the optimal sensor combinations selected by the two methods are different. The condition number from the optimal sensor location combination selected by the CN method is slightly smaller than that of the CA method.
- When the number of sensor equals to the number of unknown excitation, the condition numbers of the optimal sensor combinations selected by both of the methods are too large.

Condition Number of the Markov Parameter matrix

Demonstrate the effectiveness of the alternative CA method



The optimal sensor placement selected by CA method is the sub-optimal solution for the smallest ill-conditioning of the identification equation.

Condition number versus the order of the selected sensor location combination

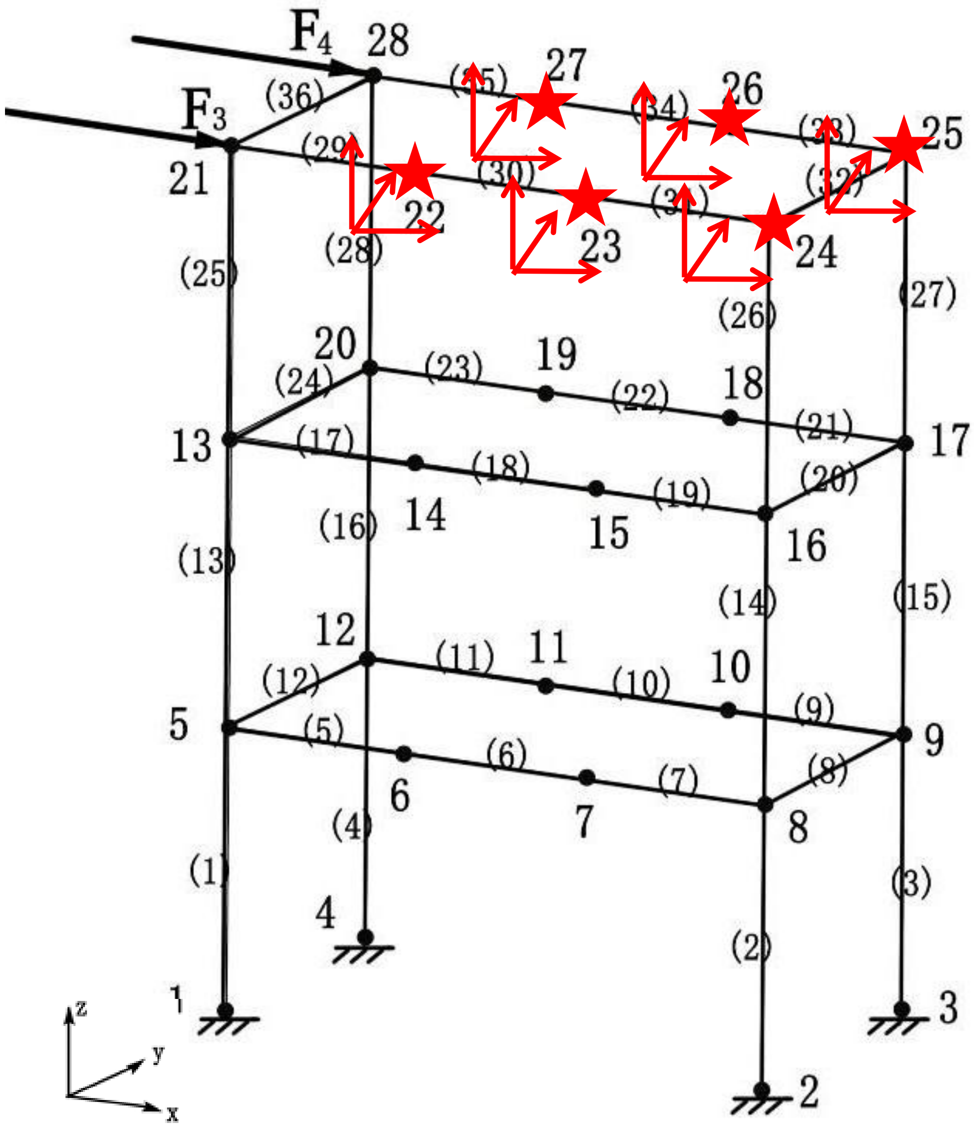
The overall trends of the two curves are the same although there are some fluctuations in the curve from the CA method. The optimal sensor combination for CA method may not always associate with the smallest condition number of the system Markov parameter matrix but it is always close to it in general.

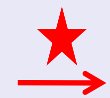
Comparison of force identification results

Type	Method	Number of sensor	Error for F_1(%)			Error for F_2(%)		
			Mean value	Standard deviation	Maximum value	Mean value	Standard deviation	Maximum value
Optimal sensor combination	CN	2	16.64	7.27	39.43	26.23	14.20	71.72
		3	8.21	2.37	17.11	12.17	3.52	25.98
		4	8.21	2.19	13.62	12.29	3.24	20.72
		5	8.87	2.15	17.19	10.98	2.62	21.68
		6	8.74	2.35	15.59	10.91	2.83	19.24
	CA	2	16.64	7.27	39.43	26.23	14.20	71.72
		3	8.21	2.37	17.11	12.17	3.52	25.98
		4	8.09	2.16	13.77	12.28	3.17	20.62
		5	7.37	1.83	13.96	10.11	2.47	19.39
		6	6.29	1.65	10.97	10.13	2.64	17.14

- In the case of 2 required sensors from the optimal sensor location combinations, the standard deviation and maximum value are too big to be acceptable. This is because the number of sensor is equal to the number of unknown force. The identification can be solved mathematically but with serious ill-conditioning with measurement noise.
- From the results with 4 or more sensors, the error of identification from the CA method is slightly smaller than that from the CN method which has a smaller condition number in the Markov parameter matrix. **This would suggest that the force identification is influenced not only by the conditioning of the Markov parameter matrix but also by the measurement noise.** The influences of noise effects can be reduced when the correlation between the measured responses is smaller.

Three-dimensional Frame structure



	Candidate sensor location, translational DOF in x direction
	Candidate sensor location, translational DOF in y direction
	Candidate sensor location, translational DOF in z direction

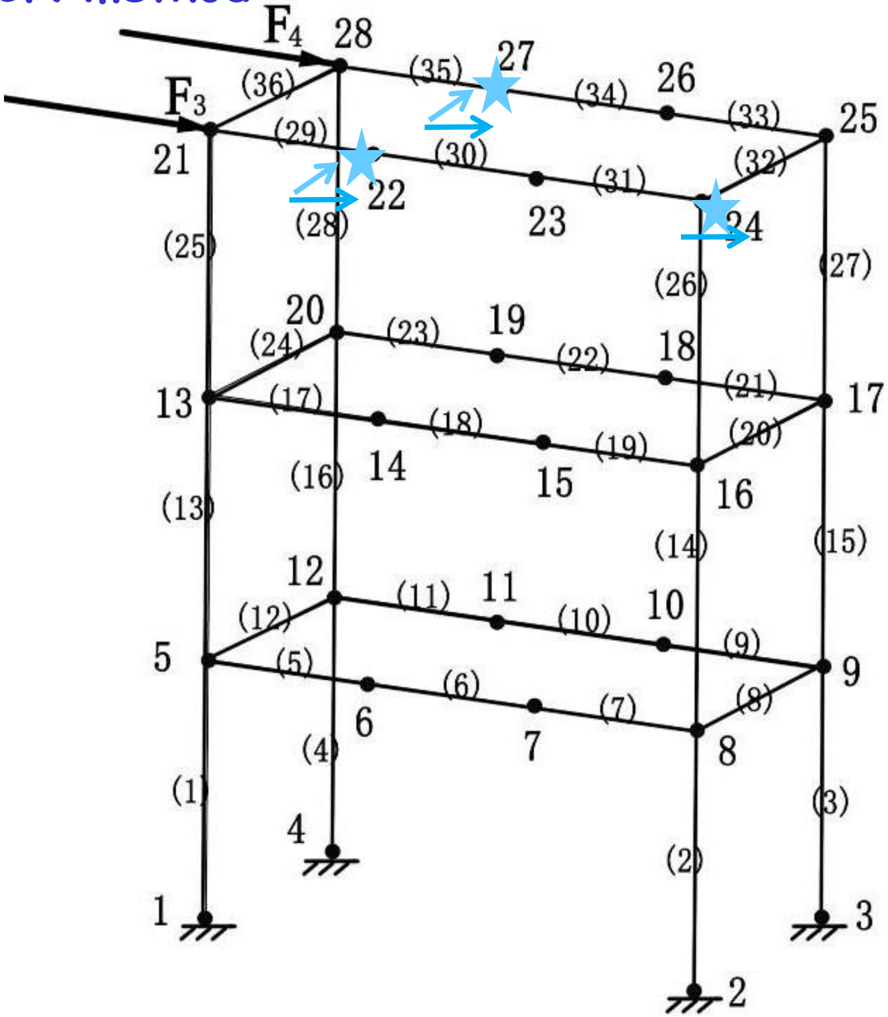
$$\mathbf{F}_3(t) = 3500\sin(3\pi t + 0.5\pi) + 2000\sin(8\pi t)\,N$$

$$\mathbf{F}_4(t) = 3000\sin(4\pi t) + 2000\sin(9\pi t + 1.4\pi)\,N$$

The computation effort required by the CN method is 116953.2 s which is more than 600 times greater than 182.27s required by the CA method.

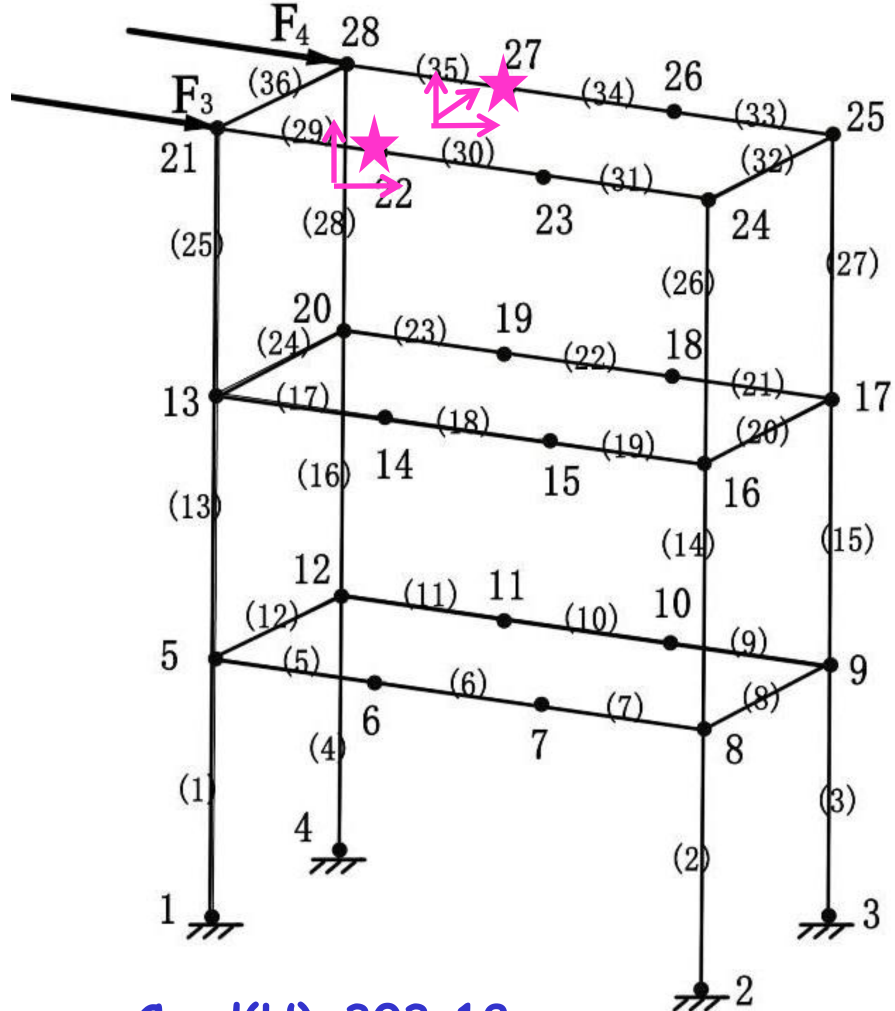
Comparison of optimal sensor placement

CN method



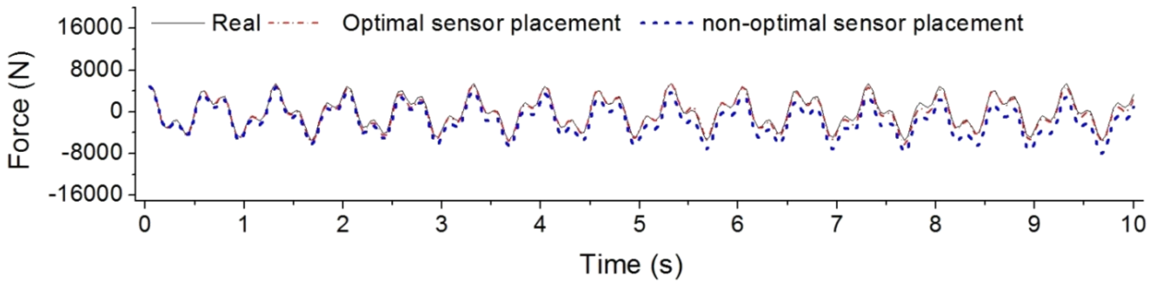
Cond(H)=294.93

CA method

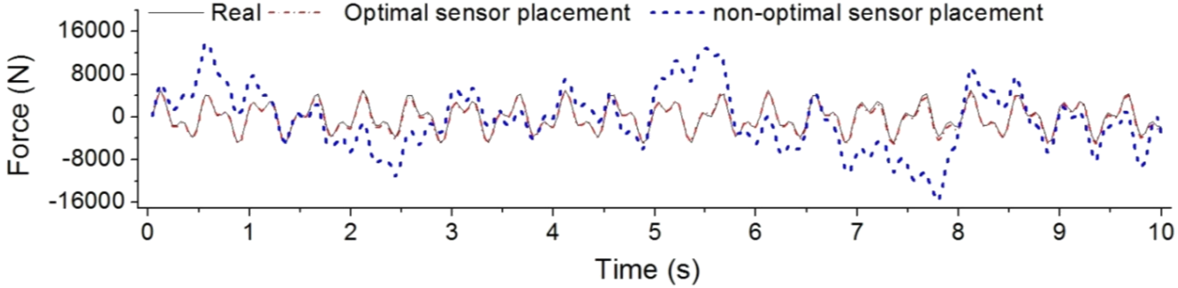


Cond(H)=303.18

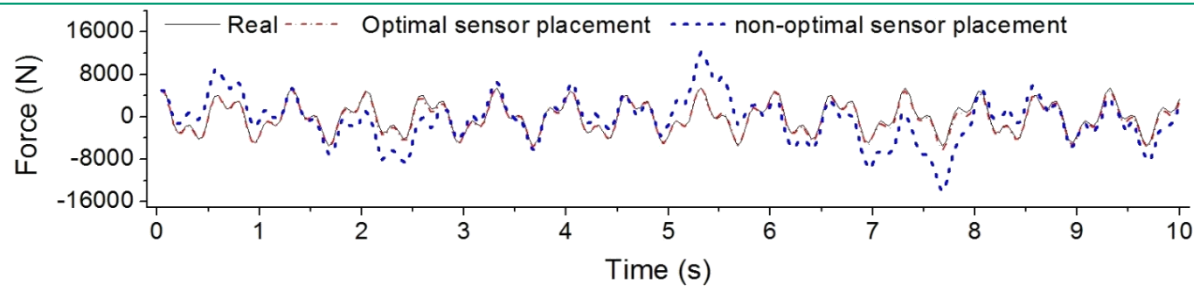
The locations in the optimal sensor combination are close to the location of external forces.



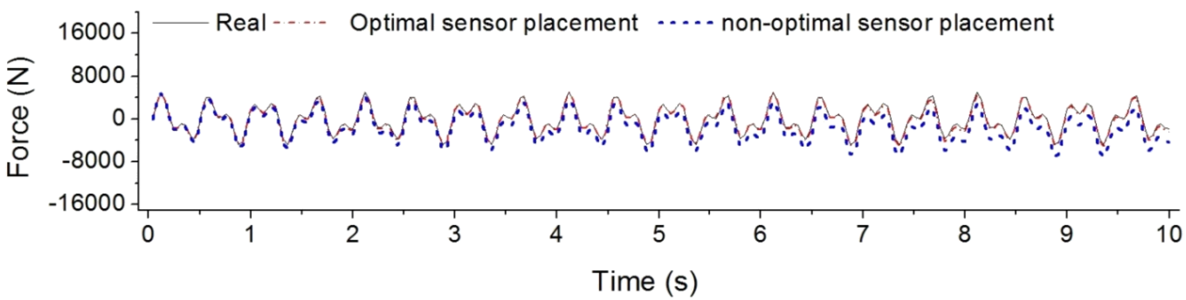
Identification error of F_3 , CN method



Identification error of F_3 , CN method



Identification error of F_4 , CA method



Identification error of F_4 , CA method

Conclusions

- Two different sensor placement methods based on conditioning analysis of the system Markov parameter matrix are presented. The first method is based on direct computation of the condition number of the matrix. Sensor location combination corresponding to the minimum condition number can be considered as the optimal sensor placement. The second method is based on correlation analysis of the Markov parameter matrix. Sensor correlation criterion is used as a measure to select the sensor locations.

- Numerical simulations show that both methods can provide consistently good sensor placements. If the sensor placement problem is small, either method can be adopted to yield satisfactory combinations of sensor locations with acceptable accuracy and computing time. However, when there are many candidate sensor combinations, the selection based on the correlation analysis has a great advantage with the computation efficiency and yet with similar accuracy of identification.

- The selection may not always associate with the smallest condition number of the system Markov parameter matrix, but it is close to it in general.